Week 7: Sound radiation, acoustic loading, directivity

Microphone and Loudspeaker Design - Level 5

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A weekly fact about Salford..!

Did you know...

- James Prescott Joule was born and raised in Salford. When he lived in the house, he conducted experiments into the nature of heat, as the plaque on the outside wall explains.

What are we covering today?

1. Sound radiation
2. Ideal piston directivity
3. Radiation impedance
4. Infinite baffle loudspeaker

Our finished circuit (for now...)

- This equivalent circuit represents a complete low frequency (lumped parameter) model of a loudspeaker driver loaded by two arbitrary acoustic impedances.
- The acoustic loading Z_{Af} and Z_{Ab} dictates whether the driver is housed in an infinite baffle, sealed or vented cabinet.

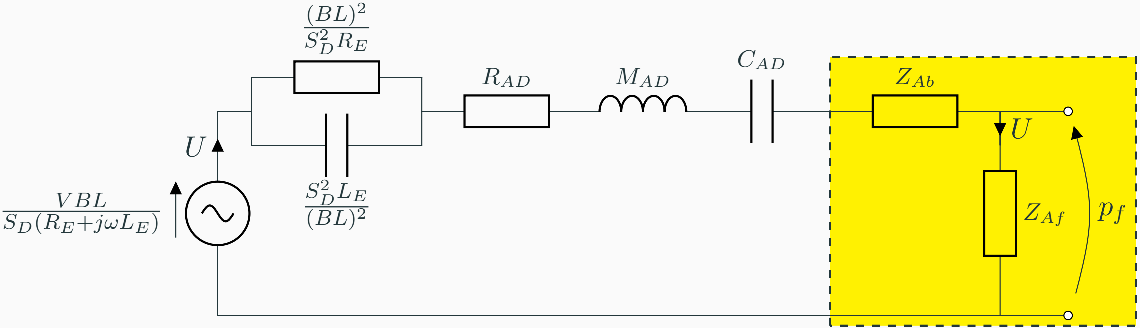


Figure 1: Equiv. impedance circuit with transformers removed and Norton's theorem applied.

Sound radiation

Monopole: the acoustic building block

- The simplest acoustic source!
- Very difficult to make a real monopole
  - At low frequencies most sound sources are approx. monopoles
- **Key features:**
- Radiates sound in all directions with equal intensity
- Surrounding pressure field depends only on distance (no angular dependence)
- Strength characterized by its volume velocity U

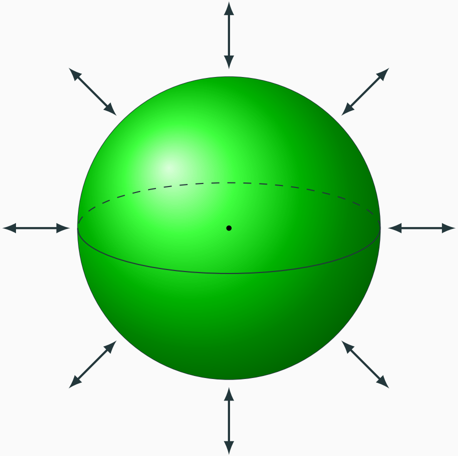


Figure 2: Monopole as a pulsating sphere with surface velocity u.

Monopole: the acoustic building block

- Pressure radiated by a monopole:

$$p(r,t) = \frac{j\rho_0 cka^2u}{r}e^{j(\omega t-kr)} \qquad (1)$$

  where ρ_0 density of air, c speed of sound, a monopole radius, k wave number, u sphere surface velocity, r distance from monopole.

- You will cover this more with Olga in semester two!

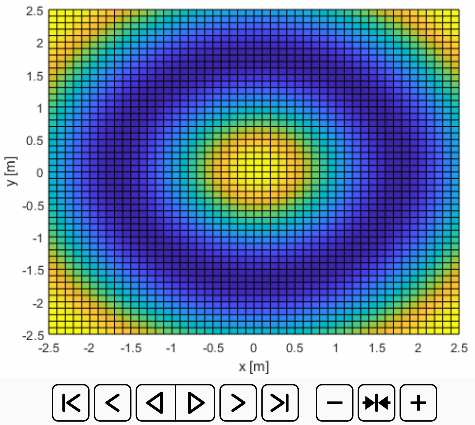


Figure 3: Radiation from a monopole

Monopole: simple radiation model

- Two important elements

$$p(r,t) = \frac{j\rho_0 c k a^2 u}{r} e^{j(\omega t - kr)} \qquad (2)$$

- The wave number $k = \omega/c$ depends on frequency - more efficient radiation at high frequencies
- The sphere volume velocity (source strength) $U = 4\pi a^2 u \rightarrow u = U/4\pi a^2$

$$p(r,t) = \frac{j\rho_0 w U}{4\pi r} e^{j(\omega t - kr)} \qquad (3)$$

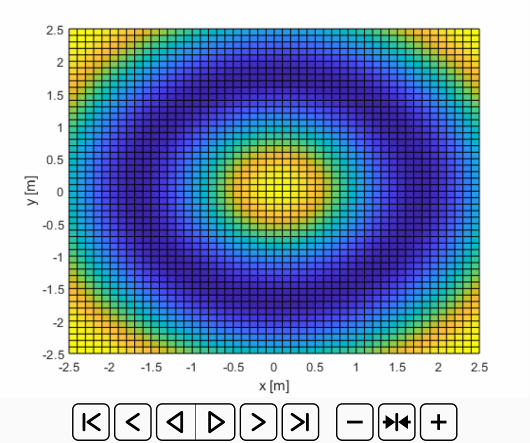


Figure 3: Radiation from a monopole

Baffled monopole: simple radiation model

- Monopole in an infinite baffle, acoustic waves are reflected in phase, radiated pressure is doubled.

$$p(r,t) = \frac{2A}{r} e^{-jkr} \tag{4}$$

$$A = \frac{j\rho_0 ckU}{4\pi} e^{j\omega t} \tag{5}$$

- At low frequencies this simple model performs remarkably well.
- At high frequencies the driver will be large compared to wave length and this model wont work...

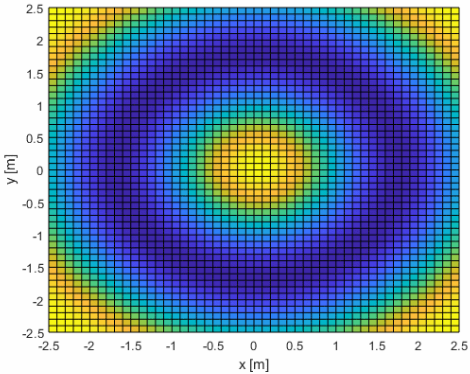


Figure 3: Radiation from a monopole

Dipole

- We can make new acoustic sources by combining monopoles...
- A dipole consists of two monopole separated by a small distance, but operating out of phase

$$p(r,t) = \frac{A_1}{r_1} e^{-jkr_1} + (-) \frac{A_2}{r_2} e^{-jkr_2} \tag{6}$$

- After some trig ($\Delta r = d\cos\theta$), and some algebra...

$$p(r,t) = \frac{A}{r} e^{-jkr} \times jkd\cos\theta \tag{7}$$

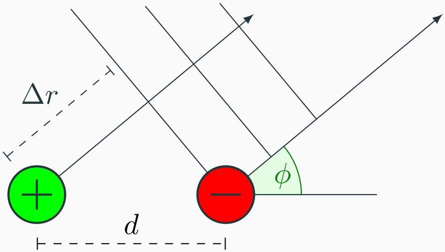


Figure 4: Dipole as a pair of monopoles radiating out of phase.

Dipole

- Difference between monopole and dipole:

$$p(r,\theta,t) = \frac{A}{r}e^{-jkr} \times \boxed{jkd\cos\theta} \qquad (8)$$

- Added frequency dependence (k), and figure of eight directivity (θ)
- Dipole is a really good model of a loudspeaker driver in free field

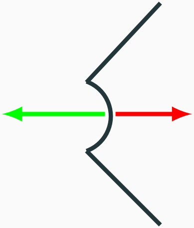


Figure 5: Dipole behaviour of free driver.

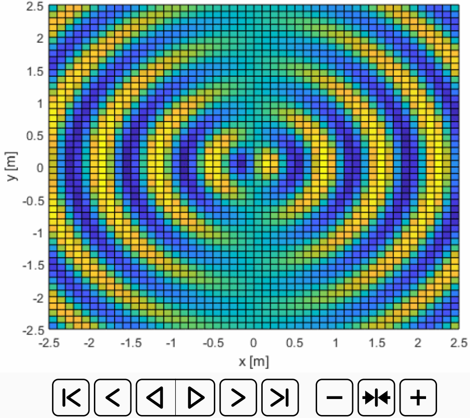


Figure 6: Radiation from a dipole

Piston

- How can we combine multiple monopoles to create a source that looks more like a loudspeaker diaphragm?
- Integrate monopole equation over circle

$$p(r,t) = \int_S \frac{A}{r} e^{-jkr} dr \tag{9}$$

- ...after some not very nice maths we get

$$p(r,\theta,t) = \frac{j\rho_0 ckU}{4\pi r} e^{j(\omega t - kr)} \left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta} \right] \tag{10}$$

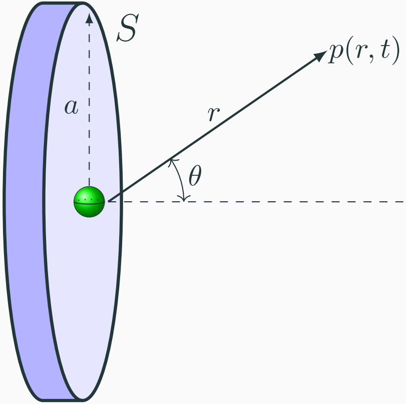


Figure 7: Piston source by integrating monopole over surface.

Piston

- Lets have a closer look

$$p(r,\theta,t) = \frac{j\rho_0 ckU}{4\pi r} e^{j(\omega t - kr)} \left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta} \right]$$

- Two parts: monopole term, directivity factor
- For constant volume velocity U the pressure increases with frequency.
- But wait, what does U look like? (from equivalent circuit) - it is resonant!

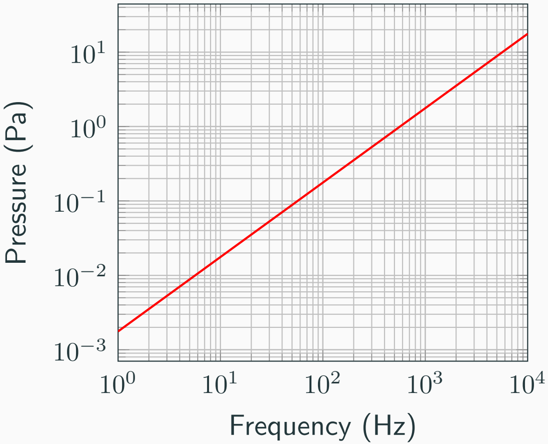


Figure 8: Monopole frequency dependence.

Piston: loudspeaker radiation

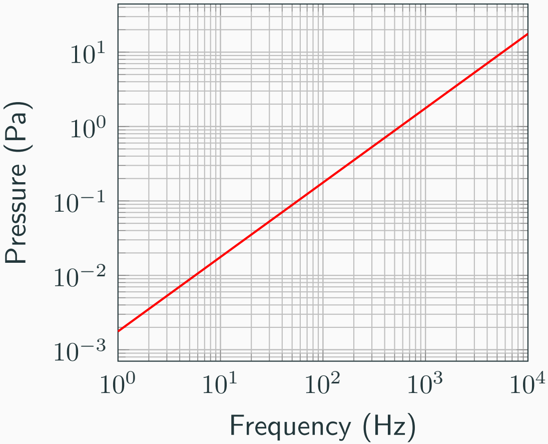


Figure 9: Monopole frequency dependence.

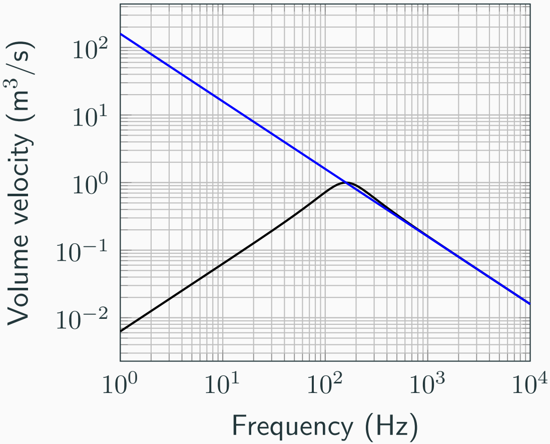


Figure 10: Resonant volume velocity.

Piston: loudspeaker radiation

- Driver is resonant, falls at -6 dB per octave...
- The pressure radiated increases at 6 dB per octave...
- **Combined effect**: flat frequency response - *only above the driver resonance though*
- **Important design concept**: make the diaphragm resonance as low as possible to extend flat region

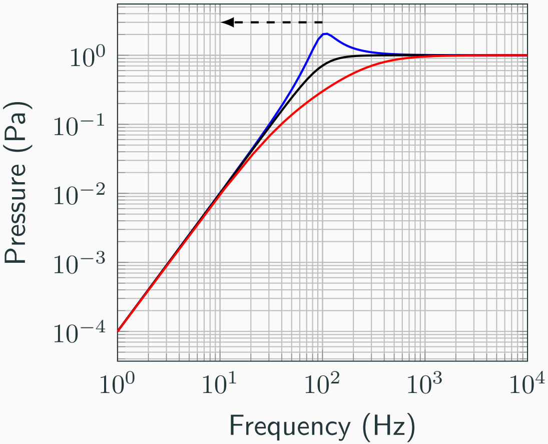


Figure 11: Radiated pressure from piston.

Ideal piston directivity

Piston

- Monopole term gives us a flat frequency response above the diaphragm resonance

$$p(r,\theta,t) = \frac{j\rho_0 ckU}{4\pi r} e^{j(\omega t - kr)} \left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta} \right]$$

- What about directivity term?
- Contains function $J_1(\quad)$ - 1st order Bessel function of the first kind
  - The canonical solutions of Bessel's differential equation
  - Represent solutions to the wave equation in cylindrical and spherical coordinates.

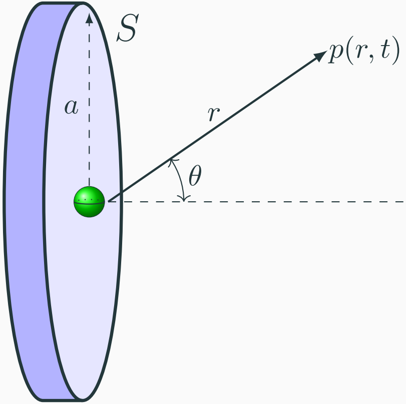


Figure 12: Piston source by integrating monopole over surface.

Piston: loudspeaker radiation

$$p(r,\theta,t) = \frac{j\rho_0 ckU}{4\pi r} e^{j(\omega t - kr)} \overbrace{\left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right]}^{\text{Directivity factor}}$$

- Bessel functions (of the first kind) look like decaying sinusoids - arise all the time when we are dealing with circular structures.
- Directivity is governed by a 'jinc' function

$$\text{jinc}(x) = \frac{2J_1(x)}{x} \xrightarrow{x=ka\sin\theta} \frac{2J_1(ka\sin\theta)}{ka\sin\theta} \tag{11}$$

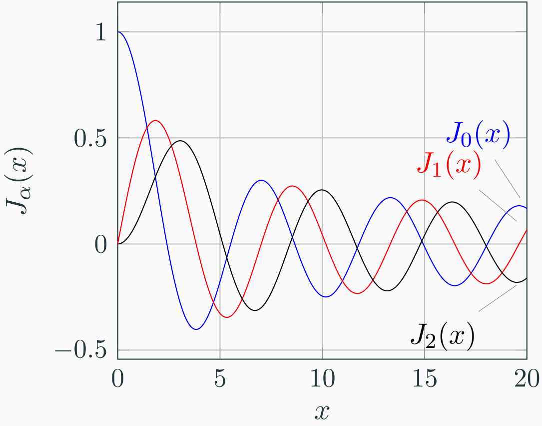


Figure 13: Behaviour of Bessel function.

Piston: angular dependence

$$\mathrm{DF}(k, a, \theta) = \frac{2J_1(ka \sin \theta)}{ka \sin \theta} \tag{12}$$

- For fixed frequency (k) and radius (a) the x-axis will vary between 0 ($\theta = 0$) and ka ($\theta = 90$)
- ka defines the upper limit (*visible region*)
- At **low frequencies**, or small piston areas, $ka << 1$, we are within the main lobe
- At **high frequencies**, or large piston area, $ka >> 1$, we see the full breadth of the main lobe, and other small lobes too.

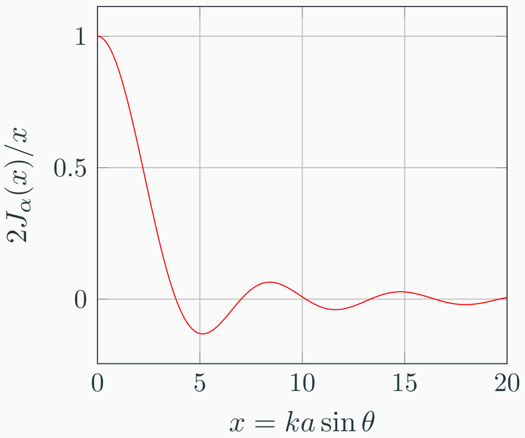


Figure 14: Behaviour of jinc function (DF).

Piston: low frequency

$$\mathsf{DF}(k, a, \theta) = \frac{2J_1(ka\sin\theta)}{ka\sin\theta}$$

- As $ka << 1$ the piston becomes approx. omnidirectional, could replace with a simple source?

$$\frac{2J_1(ka\sin\theta)}{ka\sin\theta} \xrightarrow{ka\to 0} 1 \qquad (13)$$

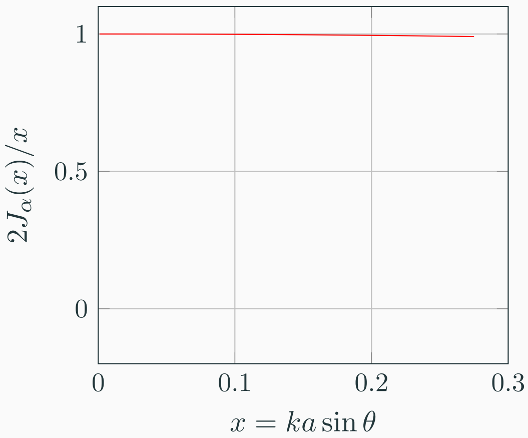


Figure 15: $f = 100$, $a = 0.15$, $ka = 0.275$.

Piston: low frequency

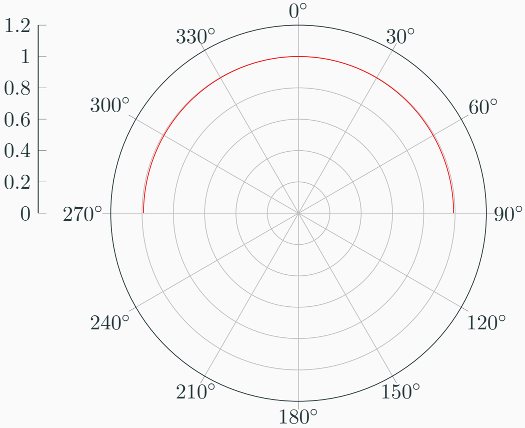


Figure 16: Polar plot

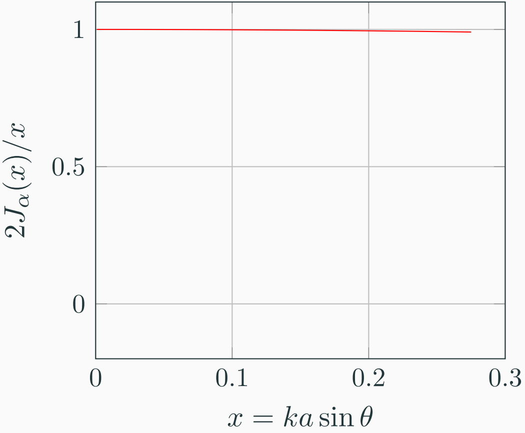


Figure 17: $f = 100$, $a = 0.15$, $ka = 0.275$.

Piston: high frequency

$$\mathsf{DF}(k, a, \theta) = \frac{2J_1(ka\sin\theta)}{ka\sin\theta}$$

- As $ka >> 1$ the piston directivity develops a narrow forward facing beam
- We also get small repeating side lobes

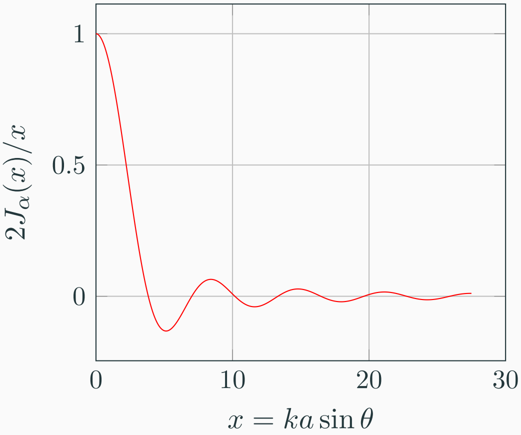


Figure 18: $f = 10000,\ a = 0.15,\ ka = 27.5.$

Piston: high frequency

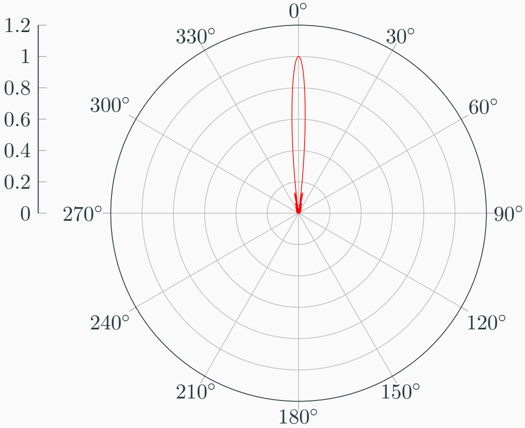


Figure 19: Polar plot

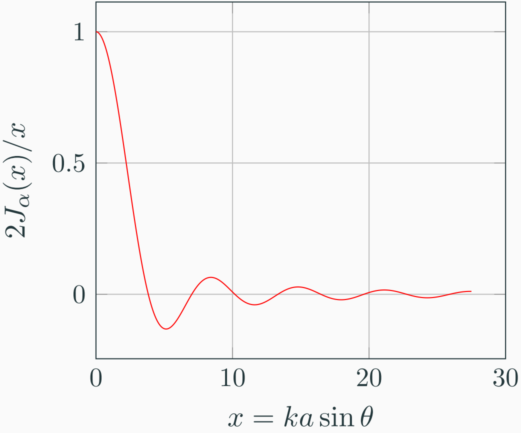


Figure 20: $f = 10000$, $a = 0.15$, $ka = 27.5$.

Piston: mid frequency

$$\mathsf{DF}(k, a, \theta) = \frac{2J_1(ka\sin\theta)}{ka\sin\theta}$$

- If $ka \sim 1$ the piston becomes mildly directional, but no side lobes
- Example: DF ≈ 0.5 at $x \approx 2.2$

$$\theta = \sin^{-1}\left(\frac{2.2}{2.75}\right) \approx 60^\circ \qquad (14)$$

- At 60° radiation is $20\log_{10}(0.5) = -6$ dB

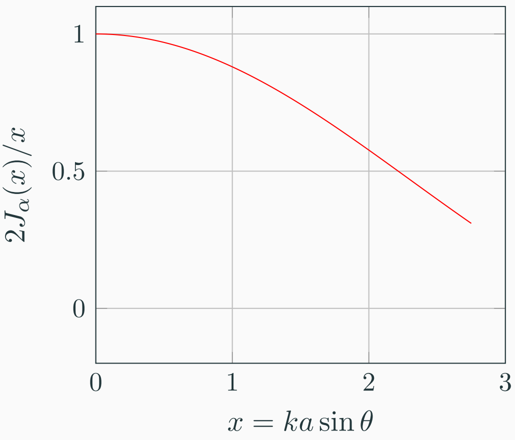


Figure 21: $f = 1000$, $a = 0.15$, $ka = 2.75$.

Piston: mid frequency

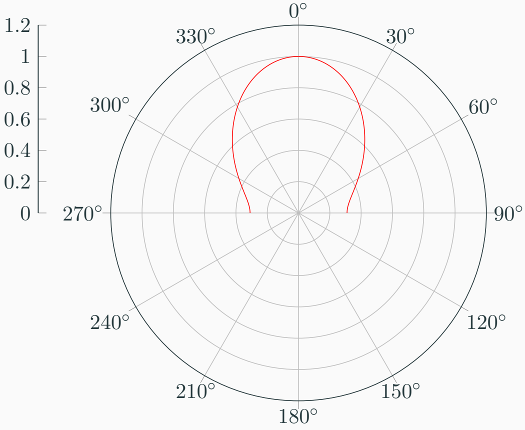


Figure 22: Polar plot

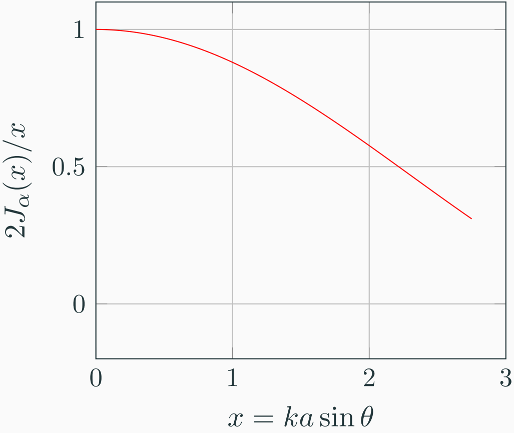


Figure 23: $f = 1000$, $a = 0.15$, $ka = 2.75$.

Piston directivity: MATLAB applet

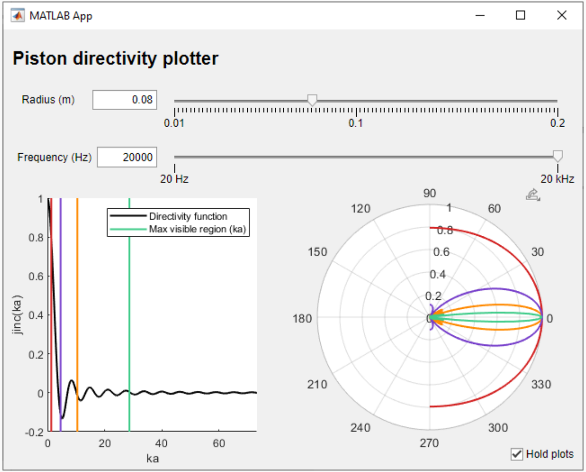


Figure 24: Click here to download MATLAB applet.

Importance of ka

- The product ka is very important! It is effectively the ratio of driver radius to wavelength:

$$ka = 2\pi \frac{a}{\lambda} \tag{15}$$

- $ka << 1$ almost omnidirectional
  - Low frequency / radius of driver small compared to wavelength
- $ka >> 1$ highly directional
  - High frequency / radius of driver large compared to wavelength

Equivalent circuit with directivity

- Combine piston model with volume velocity obtained from the equivalent circuit
- But what about the acoustic loading - the impedance provided by the air in front and behind the driver...

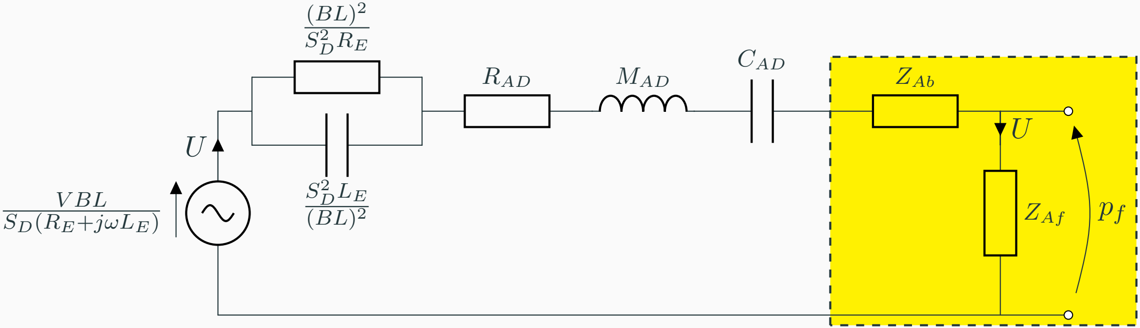


Figure 25: Equiv. impedance circuit for generic loudspeaker.

Radiation impedance

Radiation impedance

- We have considered the radiation of acoustic waves from a piston, but how does the presence of air effect the movement of the piston?
- A loudspeaker will behave differently when in a vacuum...
- A loudspeaker will experience opposition to motion due to air - this is called the **radiation impedance**
- Radiation impedance is ratio of pressure to surface velocity, times surface area:

$$Z_{rad} = \frac{p}{u} S \tag{16}$$

- For a piston we have to integrate this over its surface area (que lots of maths...)

$$Z_{rad} = \int_S \frac{p}{u} dS \tag{17}$$

Piston: radiation impedance

- In mechanical units is

$$Z_{rad} \approx \rho_0 c S \left[R_1(2ka) + jX_1(2ka)\right] \quad (18)$$

- In acoustic units is

$$Z_{rad} \approx \frac{\rho_0 c}{S} \left[R_1(2ka) + jX_1(2ka)\right] \quad (19)$$

- Approx. for $ka < 1$?

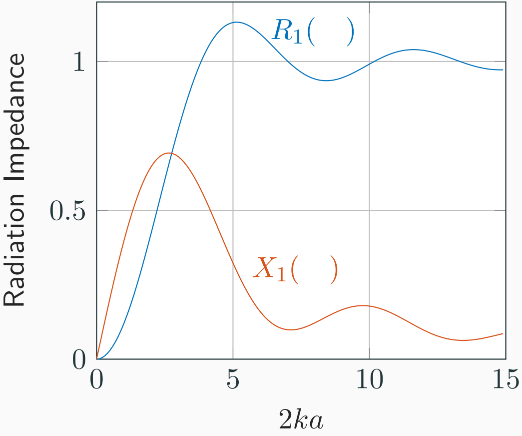


Figure 26: Piston radiation impedance

Piston: radiation impedance

- In acoustic units is

$$Z_{rad} \approx \frac{\rho_0 c}{S}\left[R_1(2ka) + jX_1(2ka)\right] \quad (20)$$

- For small frequencies and piston radii, $ka << 1$, approximates to:

$$Z_{rad} \approx \frac{1}{2}\rho_0 c(ka)^2 + j\frac{8}{3\pi}\rho_0 cSka \quad (21)$$

- Mass-like part represents mass of air 'stuck' to the piston
- Resistance-like part describes sound that is propagated

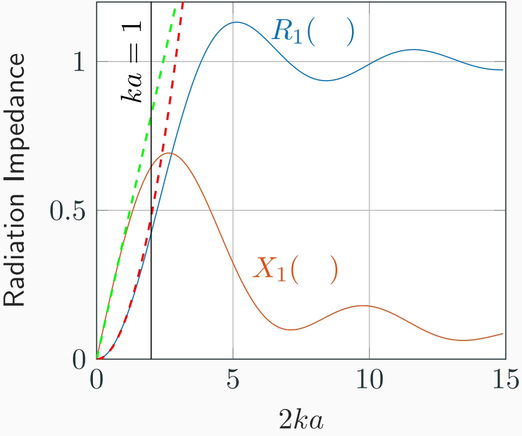


Figure 27: Piston radiation impedance

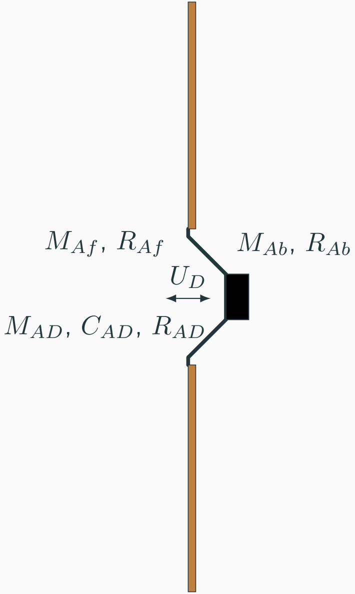
Infinite baffle loudspeaker

Infinite baffle: equivalent circuit acoustic loading

- We are ready to consider our first loudspeaker system - **an infinite baffle**
- When placed in an infinite baffle the diaphragm is loaded by the acoustic free space only
- We have derived this loading for a piston:

$$Z_{rad} \approx \frac{1}{2}\rho_0 c(ka)^2 + j\omega \frac{8}{3\pi}\rho_0 S a \qquad (22)$$

- How might we include this in our equivalent circuit model?



Infinite baffle: equivalent circuit acoustic loading

- Infinite baffle acoustic loading: $Z_{Af} = Z_{A,b} = R_A + j\omega M_A + \frac{1}{j\omega C_A}$

$$R_A = \frac{1}{2}\rho_0 c(ka)^2, \qquad M_A = \frac{8}{3\pi}\rho_0 S a, \qquad C_A = \infty \quad \left(C_A = \frac{V}{\rho_0 c^2}\right) \tag{23}$$

- Series resistor and inductor!

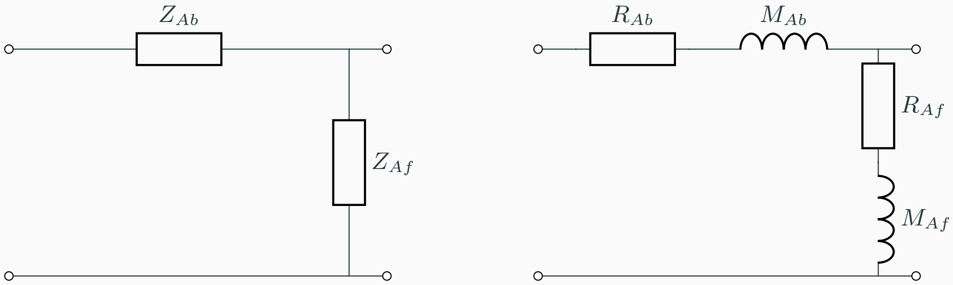


Figure 29: Equiv. circuit acoustic load for infinite baffle.

Infinite baffle: complete equivalent circuit

- We now have an equivalent circuit for a loudspeaker in an infinite baffle.
- What happens if we consider very low frequencies?
  - Parallel capacitor impedance gets very large, circuit only sees the resistor.

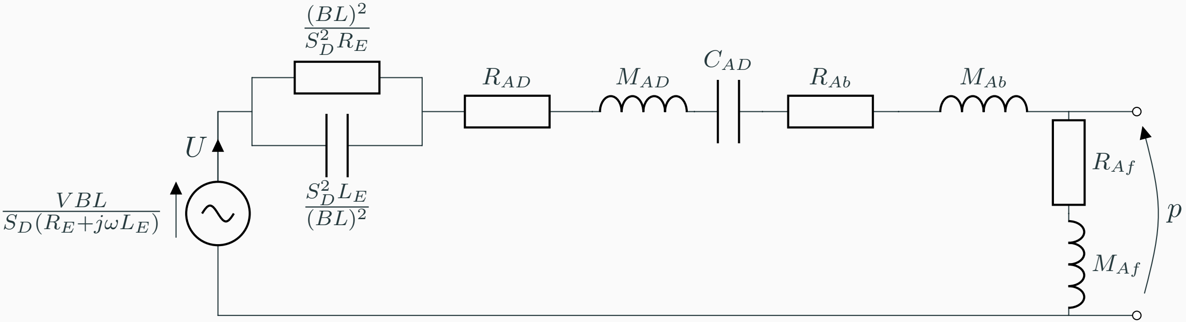


Figure 30: Equiv. impedance circuit for infinite baffle.

- Now we have a nice simple series circuit! Very easy to analyses.
- Lets group terms...

$$M_{AT} = M_{AD} + 2M_{Af} = M_{AS}, \qquad C_{AT} = C_{AD}, \qquad R_{AT} = \frac{(BL)^2}{S^2 R_E} + R_{AD} + 2R_{Af} \tag{24}$$

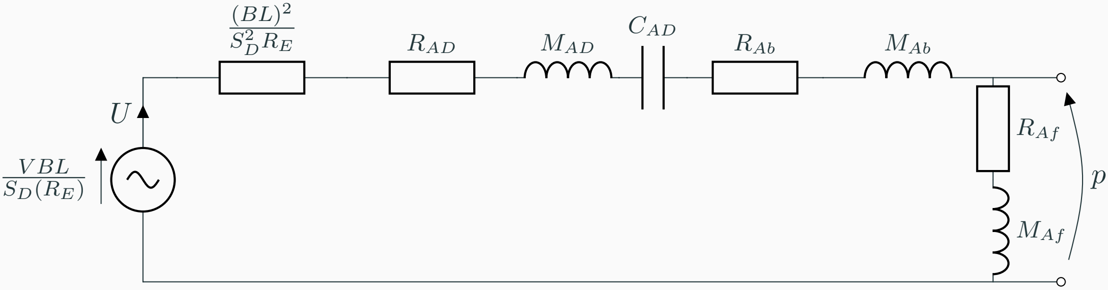


Figure 31: Equiv. impedance circuit for infinite baffle (very low freq. approx.).

Infinite baffle: simple low frequency model

- A nice simple RLC circuit - we have already analysed these!

$$M_{AT} = M_{AD} + 2M_{Af} = M_{AS}, \qquad C_{AT} = C_{AD}, \qquad R_{AT} = \frac{(BL)^2}{S^2 R_E} + R_{AD} + 2R_{Af} \tag{25}$$

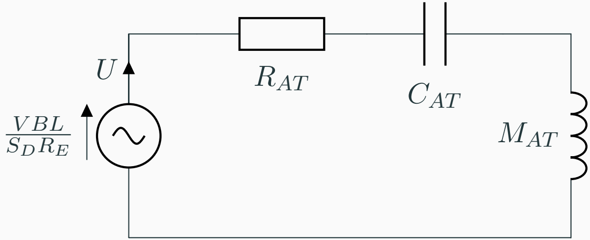


Figure 32: Equiv. impedance circuit for infinite baffle (very low freq. approx.).

Next week...

- Infinite baffle (cont.)
- Electrical impedance
- Thiele-Small parameters (i.e. what you've been measuring in your labs!)

- Reading:
  - Infinite baffle: Lecture notes, Sec 8.1
  - Electric impedance: Lecture noes, Sec 8.1.2
  - Sealed cabinet: Lecture notes, Sec. 8.2